TRANSCENDENTAL SYNTAX

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The reconciliation of the *mathematical* and *philosophical* sides of logic thanks to *informatics*.

1 — LOGIC, A MEDIATION RATIONAL/IRRATIONAL

- *Rational* from *ratio* (of a division): reduction to numbers.

  Ratiocinator: (Leibniz) verge of irrationality *(Kabbalah)*.
  Numerisation: sounds, images,... and omniscient robot.

- *Semantics* as the XXth century Kabbalah; *scientism*.

  Realism: no distinction question/answer: same *denotation*.
  Selfy: the word *pipe* refers to a PIPE, i.e., another word.
  God created the monkey in His own image.

- *Transparentism*: approach without mediation, *immediate*.

  Claim: one can *always* answer, compare and predict.
  Refuted: by *incompleteness* (Richard/Gödel/Turing).

- Logic like *police* with bad cops: no *divide* rational/irrational.

  Deductive aspects: *implication*, cheques on the unknown.
  Pact: between reason and the demons of « *déraison* ».

René Magritte (1898–1967).

2 — TRANSCENDENTALISM

- **Semantic selfy *subjectivistic* since refuses *Subject*.**

  Subject part of logic because of *logos*.
  Search for presuppositions, *conditions of possibility*.
  Analytic philosophy no presupposition; hides *prejudice*.

- ***Three lights* and *four cardinal points* against prejudice.**

  1–Answers? *Analytic:* beyond discussion, but *meaningless*.
  2–Questions? *Synthetic:* controversial, convey *meaning*.
  3–Certainty? *Reasonable;* doubts remain *legitimate*.

- **Raw/Formatted:** analytic/synthetic, untyped/typed.
  Explicit/Implicit: *a posteriori/a priori*, cut-free/deductive.

	RAW	FORMATTED
EXPLICIT	$1 - Constat$	$3 - Usine$
IMPLICIT	$2 - Performance$	$4 - Usage$

I — WHAT IS AN ANSWER?

Keywords: analytic, untyped, computational.

3 — THE ANALYTIC, A.K.A. RAW

- *Computers* yield answers, hard to ignore, even when *wrong*.

  Sever relation to question, i.e., forget *meaning*.
  Analyticity: answer *autonomous*, beyond discussion.

- *Kant:* analytic=logic, predicate part of the subject. *Outdated*.

  Modern logic contains mathematics: not *analytic*.
  Green cats are green; but *popular* democracy not *popular*.

- Dusted reading: everything *on the board*.

  Excludes any sort of external reference, in particular:
  Infinity: the « etc. » not on the board.

- *Pseudo-analyticity:* wrong claim to analyticity.

  Semantics: infinite, external, a *reasoning* about analyticity.
  Photography: problem with *offscreen*. Belongs in *usine*.
  Verbatim: really analytic, used by cowards; *meaningless*.

4 — IMPLICIT VS. EXPLICIT

- *Implicit:* what we don't have.

  Dreams, lost horizons, infinity — what we can't *finish*.
  Or want to hold off: the origin of *abstraction*.

- *Constat* vs. *performance*.

  Table of logarithms: answers hanging like smoked herrings.
  Calculator: *indirect* answer, but much more efficient.

- Opposition between two uses of ↵ *(return)* key.

  Typewriter opens new line; *incremental*.
  Computer launches program; *destructive*.

- *Give* him some tuna or *teach* him how to fish?

  Explicit answer works for a single time.
  Implicit an. general: involves *explicitation*, performance.
  Better, if pupil skilled enough; may *diverge* otherwise.

5 — QUEST OF THE EXPLICIT

- **Logicist Pavlov's dog: explicit as *semantics* of implicit.**

  Fregean opposition *denotation/sense*, contents/form, etc.
  Essentialises distinction *data/programs*.
  Same nature on the board, i.e., on *computer*.

- **Explicit as *suspended* implicit: no need to proceed *further*.**
  Chessboard: $N = 2^{64} - 1 = 18446744073709551615$.
  Cheque: *cash* (implicit) or *display* on wall (explicit)?

- ***Everything* on the board: $27 + 37 = 64$ involves $27, 37$ and:**

  Program $+$; if mistaken for $\times$, $27 + 37 = 999$, still analytic.

- ***Check* that computation done according to book (program).**

  Pavlov's dog: *meta-analyticity*. Non analytic, since *external*.
  Performance performs itself, by *matching* opposite colours.
  Explicit: uncolored (black) links, not matchable.

6 — THE GREAT ANALYTIC DIVIDE

- **The *pravdameter*, i.e., the machine to tell the truth.**

  Kabbalah, Casanova, Leibniz: unfaithful codings.
  Computers: faithful (binary) codings.
  Rational numerology: the pravdameter as the Graal of AI.

- ***Procrastination:* the pravdameter as a totalitarian fantasy.**

  Babel Library (Borges): *all* books of a given format.
  Write, characterwise, *anti-book* $\neq$ from those consulted.

- **Infinite book format *paradoxical:* Cantor vs. Turing.**

  Constative books: Library impossible, can't even *file* books.
  `Kindle`: Library exists, but some characters cannot *display*.

- ***Undecidability:* states the impossibility of *universal* answer.**

  No relation to questions; answers could be « *wrong* ».
  Cantor $\neq$ **Turing:** performance *irreducible* to constat.

7 — ONE CAN ANSWER EVERYTHING

- **The first subliminal slogan of *transparentism*.**

  X-rays of knowledge: the true reality beyond apparences.
  Realism, scientism: no doubts, down with Socrates!
  Totalitarism of NSA conspicuous in its claim to *neutrality*.

- **Undecidability: *reverse side of reality* pure fantasy, hence**

  Paradoxical: only known to gnostic sect. *Hermetism*.

- ***Hidden messages* in Nostradamus, Mallarmé, etc. baloney!**

  Hitchcock: MacGuffin, irrelevant secret message.
  Val Lewton: RKO producer, *suggestion* stronger than *vision*.

- **The *topsy-turvied* reverse « the first will be last ».**

  Escher: icon of « reverse side ». Gödel-Escher-Bach.
  Nonsensical view of logic: the man *always* telling lies.
  Superficial and transparentist: involves a *pravdameter*.

8 — PURE λ-CALCULUS

- **Best approximation to analyticity: *untyped*, no commitment.**

  Syntax: *terms* $x, \lambda x t, (t)u$.

  Constative: *normal* terms.

  Performance *rewriting* $(\lambda x t)u \rightsquigarrow t[u/x]$.

- ***Knitting*** **expressed by structural properties.**

  Church-Rosser redexes in two colours.

  Three performances equivalent.

- ***Forgetful*** **functor from typed (synthetic) systems (e.g., $\mathbb{F}$).**

  Church-Rosser: compositionality of $\Rightarrow$ *(associativity)*.

- **Limitations:**

  Externality of performance: rewriting *redex* $\rightsquigarrow$ *contractum*.

  Functional commitment; unfit for parallelism.

  Non linearity: unfit for non-determinism.

9 — UNIFICATION AND MATCHING

- **Originates in Herbrand 1930; sort of analytic η-expansion.**

  Identity $A \maltese B \vdash A \maltese B$ same as identities $A \vdash A, B \vdash B$.
  Wire splits spontaneously into *subwires*.

- **Functional *terms:* wires. *Variables:* potential subwires.**

  Subwires activated by *matchings* $t = u$.
  Example: $a * y = x * b$: common subwire $a * b$.
  Example: $a * y = b * x$ don't match, matching *fails*.

- ***Unification:* search for *most general unifier* θ_0.**

  Unifier of t, u: substitution θ s.t. $t\theta = u\theta$.
  M.g.u. θ_0: any unifier θ uniquely writes as $\theta = \theta_0\theta'$.
  Unifiers for $a * y = a * x$, the $\theta_t(x) = \theta_t(y) = t$, m.g.u. θ_z.
  Matching: dynamic intersection $t \cap u$. *Distinct* variables.
  Failure: no unifier; not matchable = *disjoint*.

10 — STARS AND CONSTELLATIONS

- *Star:* $n \neq 0$ terms *(rays)* with exactly the same variables.

  Disjoint: rays pairwise not *matchable*.

  Substitution: $[\![\, t_1, \ldots, t_n \,]\!]\theta := [\![\, t_1\theta, \ldots, t_n\theta \,]\!]$ still a star.

- *Constellation:* finite set of stars.

  Bound variables, i.e., local to each star.

  Rays of the (stars of the) constellation pairwise disjoint.

- *Colours:* just a convenience, unary function letters.

  Disjoint: come by *complementary* pairs.

  Pairs: `green`/`magenta`, `red`/`cyan`, `blue`/`yellow`.

- Colours responsible for divide *constat/performance*.

  Constative constellation: in black (no colour).

  Performance: elimination of colour, normalisation.

  GoI: analytic substrate of synthetic *cut-elimination*.

11 — STRONG NORMALISATION

- *Diagrams* of constellation: *tree* (connected/acyclic graph).

  Vertices: stars (with repetitions). Infinitely many diagrams.
  Edges: formal equalities $t = u, t = u, t = u$.

- *Actualisation* of a diagram:

  Match underlying terms: $t = u$ becomes $t\theta = u\theta$.
  Failure of most actualisations; diagram *correct* otherwise.

- *Strong normalisation:* knitting constat/performance.

  1–Finiteness: all diagrams of size N, hence $\geq N$ fail.
  Excludes $[\![\, x, x \,]\!]$. *Undecidability:* no way to predict N.
  2–Openness: no *closed* correct diagram (with no *free* ray).

- *Residual* star of correct diagram: its actualised *free* rays.

  Normal form: constellation of *uncoloured* residual stars.
  Church-Rosser: two pairs of complementary colours.

12 — NON-DETERMINISM

- ***Non-determinism*** in constellations allows matching rays.

  Resolution: stars $\Gamma \vdash A$ or $\Gamma \vdash A$: a fine mess.
  Control: tries to fix bad analyticity, e.g., *multiple matchings*.
  PROLOG: *analytic* mingled with *synthetic*, logic: fails.
  Declarative programming: similar to *analytic philosophy*.

- Same problem with π-*calculi*.

  Hesitate: parallel λ-calculus or cheap linear logic?

- Matching rays can only represent *Alzheimer*, NL-style.

  Coordination: necessary in NP-style *(satisfiability).*

- ***Non-deterministic*** constellation:

  Liberalised: matching rays allowed.
  Coherence: $\mathcal{S} \ddagger \mathcal{T}$: *forbidden* substitutions.
  Strong normalisation: *self-incoherent* diagrams fail.

13 — PARALLEL UNIVERSES

- *Church-Rosser:* takes account of all *parallel* computations.

- Knitting with *usine:* one should take care of *additives*.

  $A \& B$: choose between « parallel universes » A/B.
  Freshness: how do I know that my choice is not *biased*?
  If already in universe A, I cannot see alternative B.
  S-F analogue: movies style *The matrix*.

- Herbrand: *formal* function $f(t)$, a variable unknown to t.

  Herbrand boolean $\eta_{\mathcal{S}}$ indexed by a substar of some $\mathcal{T}$.
  Normalisation induces dynamic modification of booleans.
  Evolution of $\mathcal{T}$ into $\mathcal{T}'$ induces parallel evolution $\eta_{\mathcal{S}} \rightsquigarrow \eta_{\mathcal{S}'}$.

- $\eta_{\mathcal{A}\&\mathcal{B}}$: boolean living « outside » A/B. Chooses A.

  Cancellation with $\neg\eta_{\mathcal{A}\&\mathcal{B}}$: only if behave in *same* way.
  Arrival in $A \& B$: not influenced by dichotimy A/B.

II — WHAT IS A QUESTION?

Keywords: synthetic, typed, logical.

The Ouija board (~ 1890).

14 — BEYOND TRUTH AND FALSITY

- *Ouija* board; talks with « **spook** »; which answers by *beep*.

  Irrational, but why? Think of an *Ipad;* what means *beep*?

  Polygraph: does not quite mean « **liar** », only « **it matters** ».

  Locative: no real contents, beyond discussion: « **touché** ».

- « **Do you know what time it is? – Yes.** » Unsatisfactory.

  Witness: would-be « **proof** » of answer *yes*.

  Doubt: false witness. No benchmark (pravdameter) *yes/no*.

  Witness fails to convince; need *interrogation* process.

- « **Did you bring a DVD reader? — Yes, see.** »

  Interrogation: Feed *reader* with DVD, use remote control.

  Remote vs. menus *language-free* dialogue.

  Witness convinces if movie actually played.

  Dialectics *witness* (reader) vs. *ordeal* (rigid DVD).

15 — HEGELIAN, A.K.A. LINEAR, NEGATION

- **Relation *witness/ordeal* symmetrical: mutual agreement.**

  Negation $\sim Q$ **replaces answers to** Q **with its ordeals.**

  Witnesses of $\sim Q$ **are the constraints on** Q.

  Hegel: contradictory foundations: Q **rests on « contrary ».**

- **Not to be confused with usual negation** $\neg A$.

  Witness: negation problematic; there is no « non-witness ».

  $\neg$ **not *involutive*. Weaker, more expedient than** $\sim$.

  The Godfather (1972): « a proposal that you cannot refuse ».

- **According to semantic pleonasm, negation** $\neg$ *refutes*.

  Hegelian (a.k.a. linear) negation *recuses*.

  Affaire Dreyfus: « la question ne sera posée ».

  Hegelian negation is a sort of *normative*, deontic prison.

  Format a.k.a. synthetic. Formation (> 0) vs. formatting (< 0).

16 — JUDGES WILL BE JUDGED

- **Possible *dissensus:* reader cannot read disk.**

  Bad reader? Or defective DVD? No simple way to tell.

  Sampling: restrict to generic ordeals, a finite *gabarit*.

  Reader-test and DVD-test always *accept* each other.

- ***Laxism* (Volkswagen): *tested* reader may refuse *tested* DVD.**

  Production $\neq$ consumption: negated by semantic *prejudice*.

  Language as well, except *ox/beef*, *calf/veal*, *sheep/mutton*.

- ***Usine:* sense as question *(gabarit)*. *Proof-nets*, *cut-free*.**

  A posteriori: experimental, everything checked.

  Almost analytic: but for questionable *choice* of tests.

- ***Usage:* sense as use (Wittgenstein). *Indirect* answers.**

  Implication $Q \Rightarrow R$: question R reduced to Q.

  Cut: answers to $Q \Rightarrow R$ and to Q perform into answer to R.

  Sampling changed by implication. Cannot stay within *usine*.

17 — THE ARCHITECTURE OF THOUGHT

- *Richard's paradox* (1905); inspired Gödel's theorem.

  Smallest integer not definable in ≤ 20 words.

  Fixed with rigourous version DEFINABLE. Not a DEFINITION.

  Refutes *qualunquism*, the analytic, « anti-format » ideology.

- Charybdis/Scylla: *inconsistency* vs. *incompleteness*.

  Inconsistency: format too *laxist* (original paradox).

  Incompleteness: format too *repressive* (« fixed » version).

- Three ages of living formats: *young, senile, post-mortem*.

  Young: protection, e.g., informatic *extensions*.

  Mathematics: stimulates, structures. Groups, morphisms.

  Senile: repressive. The russian *Tchin*. Academism.

  Apple: more and more repressive, hence the *jailbreak*.

  Post-mortem: play on the format, « second degré ».

  Has been: outed from rewarding format.

18 — ONE CAN COMPARE EVERYTHING

- **Second subliminal slogan of *transparentism*.**

  Unidimensionality: unique number, bibliomtery, QI, etc.
  Global comparison: impossible, like in *Jan-Ken-Pon*.
  Best all-times movies: reflects the jury, e.g., Brussels 1958.

- ***Complotism*, the unidimensional version of topsy-turvism:**

  Axis of Evil: Saddam, Kim, ben Laden, meet underground.
  Void of contents: only purpose seems to be *abstract* Evil.

- ***Transcription:* supposes unidimensionality.**

  Numerology: Casanova, etc. Fails even if made faithful.
  Sound to image: *Fantasia* (1940) not convincing.
  Image to sound: Xenakis (yields rumbles).
  Sound to taste: piano à cocktails (Boris Vian).
  Language to music: BACH, DSCH for Shostakovich; dubious.
  Translation: cannot render nuances in foreign language.

19 — QUALUNQUISM

- **Format is half good, half bad; unless we try to *break* it.**

  Essentialism: *conservative*, everything in its place.
  Existentialism: *protest*, counter-power.
  Complementarity: all formats *injust*; but we *need* one.

- ***L'uomo qualunque:*** (ordinary man) neo-fascist party (1946).

  Populism: le Pen, Sarkozy: *down* with politics and Justice!
  Great Leap Forward (1958): production of qualunquist steel.

- **Analytic philosophy as *tabula rasa* (Russell $\sim$ 1925).**

  Down with concepts: not rigourous enough! Use logic.
  Transcription problematic: how do you say « God » in logic?
  Logic not analytic. Disputable, esp. dubious *predicate* part.

- ***Declarative* (logic) programming: down with algorithmics!**

  Ad hoc: « control » ($\sim$ philosophical « logics », the *Führer*).

20 — MULTIPLICATIVE PROOF-NETS

- **Function symbols** $\mathtt{l}, \mathtt{r}, \mathtt{g}$ **(0-ary),** $\cdot$ **binary.**

  To each proposition A **associate** *location* $p_A(x)$.

  To each proof π **associate** *vehicle* $\pi^\bullet$.

  Identity axiom $\vdash A, \sim A$: $\pi^\bullet := [\![\, p_A(x), p_{\sim A}(x) \,]\!]$.

- $p_A(x) := p_{A \boxtimes B}(\mathtt{l} \cdot x), p_B(x) := p_{A \boxtimes B}(\mathtt{r} \cdot x)$ ($\boxtimes = \otimes, \mathbin{\bindnasrepma}, \ldots$)

  $\mathbin{\bindnasrepma}$**-rule: if** π **comes from** ν **of** $\vdash \Gamma, A, B$, $\pi^\bullet := \nu^\bullet$.

  $\otimes$**-rule: if** π **from** ν, μ **of** $\vdash \Gamma, A, \vdash \Delta, B$, **then** $\pi^\bullet := \nu^\bullet + \mu^\bullet$.

- *Ordeals:* $q_A(x) := p_A(\mathtt{g} \cdot x)$; **the** $q_A(x)$ **pairwise** *disjoint*.

  Conclusions: $\color{green}{\texttt{green}}/\texttt{black}$, **premises:** $\color{magenta}{\texttt{magenta}}/\color{olive}{\texttt{yellow}}$.

- $\texttt{LEGO}$ *bricks:* **Literals:** $[\![\, \frac{p_A(x)}{q_A(x)} \,]\!]$; **conclusion** $A \in \Gamma$: $[\![\, \frac{q_A(x)}{p_A(x)} \,]\!]$.

  $\otimes$**-link:** $[\![\, \frac{q_A(x), q_B(x)}{q_{A \otimes B}(x)} \,]\!]$.

  $\mathbin{\bindnasrepma}$**-links:** $[\![\, \frac{q_A(x)}{q_{A \mathbin{\bindnasrepma} B}(x)} \,]\!] + [\![\, \frac{q_B(x)}{} \,]\!]$ **or** $[\![\, \frac{q_A(x)}{} \,]\!] + [\![\, \frac{q_B(x)}{q_{A \mathbin{\bindnasrepma} B}(x)} \,]\!]$.

21 — CORRECTNESS

- *Gabarit:* **all ordeals obtained by** *switching* **the $\mathscr{B}$-links.**

  Vehicles coloured in `blue`**.**

  Correctness: $\mathcal{V} + \mathcal{O}$ **strongly normalises into**

  Normal form: $[\![\, p_\Gamma(x) \,]\!] := [\![\, \{p_A(x); A \in \Gamma\} \,]\!]$.

- η**-expansion: identity links on literals. Criterion insensitive.**

- *Herbrand:* **existentials as functions of universals** $\vec{y} = \vec{t}[\vec{x}]$.

  $x := f(y)$ **as independence of** $y = t$ **from** x**, i.e.,** $\exists y \forall x$.

- $X\,(\sim X)$ **must be paired; not with** $X, Y, \sim Y\,(\sim X, Y, \sim Y)$.

  Essentialism: complementarity of *names***.**

  Literal $X, \sim X$**: occ. of** *universally* **quantified variable** $\forall X$**.**

  Cancelling ordeal: special kind bound to normalise to $\emptyset$**.**

  Switching: select a literal in all pairs, $\sim X, \sim Y, Z$**.**

  Sum of all: $[\![\, \frac{q_A(\mathbf{l}\cdot x),\, q_A(\mathbf{r}\cdot x)}{} \,]\!]$ **when literal** A **is selected.**

22 — THE CUT RULE

- **_Lewis Carroll (1893):_ cut identical to conclusion $A \otimes \sim A$.**

  Cut conclusion with $A \multimap A$.

  Replace cut with $(A \multimap A) \otimes A \otimes \sim A$, etc.

  Zenon: should be the same as Achilles vs. Tortoise.

  No paradox, just stupidity: Achilles runs in _wrong_ direction.

- **_Cut:_ conclusion $[A \otimes \sim A]$. Predicts erasure, _a priori_, usage.**

  Performance: vehicle $\mathcal{V}$ in `blue` and `red` (for $p_A, p_{\sim A}$). Add

  Feedback: $\mathcal{F} := [\![\frac{p_A(x), p_{\sim A}(x)}{}]\!]$.

  Elimination: from the $\mathcal{V} + \mathcal{O}_{A \otimes \sim A} + \mathcal{O}$ to the $\mathcal{V} + \mathcal{F} + \mathcal{O}$.

- **_Church-Rosser:_ use two pairs of colours.**

  Cut-elimination: adequation usine/usage.

  Knitting: compositionality, BHK.

- **Exponentials: will involve _hidden cuts_ $[\underline{A \otimes \sim A}]$.**

23 — IMPOSSIBLE CONNECTIVES

- **Operations not central and *poorly knitted*.**

  Exponentials: $!A, ?A$.

  Intuitionistic disjunction: $!A \oplus !B$; *commutative* cuts.

  Multiplicative neutrals: $1, \bot$.

- **These connectives only acceptable as *second-order* ones.**

  Exponentials: $!A := \forall X((A \Rightarrow X) \multimap X)$.

  Int. disj.: $!A \oplus !B := \forall X((A \Rightarrow X) \multimap ((B \Rightarrow X) \multimap X))$.

  Multiplicative neutrals: $1 := \forall X(X \Rightarrow X)$.

- **Basic problem: *weakening* impossible.**

  From Γ: no way to derive Γ, A for any A.

  Want of physical connection.

  Hidden conclusion: $\Gamma, \underline{\Delta}$.

  Ordeal: $[\![\, \frac{q_A(x)}{} \,]\!]$ when $A \in \Delta$ hidden (variant below).

24 — EXPONENTIALS REVISITED

- **Revert to *intuitionistic* implication... Not quite.**

  Bang! $A \oslash B := {!A} \otimes B$.

  Why not? $A \ltimes B := {?A} \,\invamp\, B$.

- **Vehicles: auxiliary variable for *duplication:* $p_A(x \cdot y)$.**

  Dereliction: $\vdash \Gamma, \underline{\Delta}, \underline{A}$ **from** $\vdash \Gamma, \underline{\Delta}, A$: $p_A(-) \rightsquigarrow p_A(- \cdot \mathrm{d})$.

  Weakening: **no change.**

  Contraction: $\vdash \Gamma, \underline{\Delta}, \underline{A}$ **from** $\vdash \Gamma, \underline{\Delta}, \underline{A}', \underline{A}''$:
  $p_{A'}(- \cdot -), p_{A''}(- \cdot -) \rightsquigarrow p_A(- \cdot (\mathtt{l} \cdot -)), p_A(- \cdot (\mathtt{r} \cdot -))$.

  $\ltimes$**-rule:** $\vdash \Gamma, \underline{\Delta}, A \ltimes B$ **from** $\vdash \Gamma, \underline{\Delta}, \underline{A}, B$:
  $p_A(-) \rightsquigarrow p_{A \ltimes B}(\mathtt{l} \cdot -)$ **and** $p_B(-)) \rightsquigarrow p_{A \ltimes B}(\mathtt{r} \cdot -)$.

  $\oslash$**-rule:** $\vdash \Gamma', \underline{\Delta}, \underline{\Delta}', A \oslash B$ **from** $\vdash \underline{\Delta}, A$ **and** $\vdash \Gamma', \underline{\Delta}', B$:
  $p_A(-) \rightsquigarrow p_{A \ltimes B}(\mathtt{l} \cdot (- \cdot y))$ **and** $p_B(-)) \rightsquigarrow p_{A \oslash B}(\mathtt{r} \cdot -)$.

  Homogeneise **to take care of auxiliary variable and sum up.**

25 — EXPONENTIAL CRITERION

- ***Proof-nets:*** **auxiliary variable induces problems.**

  Normal form **of** $\mathcal{V} + \mathcal{O}$ **of the form** $[\![\, p_\Gamma(x) + p_\Delta(x) \cdot T \,]\!]$.
  No way **to foretell** T **(complex weakening/contraction).**
  Transcendalism: **weakening/contraction not part of answer.**

- **Criterion involves** ***non-determinism.***

  $A \oslash B$**:** $[\![\, \dfrac{q_A(x) \cdot x, q_B(x)}{q_{A \oslash B}(x)} \,]\!]$ **and** $[\![\, \dfrac{q_A(x) \cdot 1, q_B(x)}{q_{A \oslash B}(x)} \,]\!]$.

  Solution $q_A(x).t$ **with** t **unifying with both of** $x, 1$**:** $t = y$.

  $A \ltimes B$ **sort of** $\invamp$ **without left switching.**

  $\ltimes_R$**:** $[\![\, \dfrac{q_B(x)}{q_{A \ltimes B}(x)} \,]\!] + [\![\, \dfrac{q_A(x) \cdot y}{} \,]\!] + [\![\, \dfrac{q_A(x') \cdot y'}{} \,]\!]$ $(x \neq x')$.

  Solution $q_A(t_i).u_i$ **would produce duplicate if** $t_i \neq x$.

  $\ltimes_L$**:** $[\![\, \dfrac{q_A(x) \cdot y}{q_{A \ltimes B}(x \cdot y)} \,]\!]$ **(cancelling).**

  Impossibility **to reach premise** A **« from below ».**

III — WHAT CONVEYS CERTAINTY?

Keywords: derealism, epidictic, épure.

26 — HILBERT OUT OF FOCUS

- *Axiomatics:* in modern greek, *officer*, not quite logical!

  XIX$^{\text{th}}$ century: axioms + *Modus Ponens* (usage), no usine.
  Mistakes: located in « false » axioms; no *pravdameter*.

- Sort of usine: *limited* questions $2 + 2 = 4$ or $2 + 2 = 5$.

  Consistency: axioms should not yield *incorrect* $2 + 2 = 5$.
  Kant « fixed » by Hilbert: *consistency* of presuppositions.
  Scientistic *self*-justification of science.

- *So far so good:* consistency not analytic (not performative).

  Incompleteness: neither *checkable* nor *provable*.
  Inconsistency analytic : performative, *expansive*.

- *Confidence* not ensured by consistency.

  Inconsistency consistent? *Indirect* proof procrastinates.
  Never seen never taken: a sort of logical *dismissal*.

27 — ON THE TRAIL OF THE DOUBT

- **Axiomatic** *smoothing:* **tree-like form** *inexpressive.*

  Realism: *Modus Ponens* **preserves** *ethereal* **truth.**
  Usine: **axioms can usually be** *checked.*
  Modus Ponens **problematic, involves change of** *gabarit.*

- *The fly (1986):* **neither man nor fly, nor both!** *Mix* **man+fly.**

  Imbrication: $Q \multimap R$ **imbricates questions** $\sim Q$ **and** R.
  Sequent $\vdash Q, R$ **imbricated questions.**
  Extension wire $\vdash \sim Q, Q$ **production/consumption of 127V.**

- « **The medium is the message** »: **sense is form, shape.**

  Proof-nets **trees imbricated through** *paired leaves.*
  Travel **not tree-like; conveys actual** *semantic-free* **meaning.**

- *Desimbrication.* **Recover man from mix** *man+fly.*

  Lewis Carroll **imbricates! Need** *Cut* **with** « **anti-fly** ».

28 — RIGHTS AND DUTIES

- **Cut involves a *performance*; may diverge (procrastinate).**

  Laxist *gabarits:* Volkswagen.

- **Mismatch usine/usage (*Prawitz:* introduction/elimination).**

  Usine: the *right* to use a name.
  Usage: the corresponding *duties*.

- **Mismatch comes from *incomplete* gabarits.**

  Perfect case: (multiplicatives, etc.) *possible* completion.
  Imperfect case: (exponentials + second order) *impossible*.

- ***Popper:* use incomplete gabarits; « so far so good ».**

  Fitted for *medicine*, since non deductive.
  Empirism: restricted to *reproduction*.
  Unfitted for prevision, deduction. « Butterfy effect ».
  Gabarits deeply altered by *indirect*, deductive answering.

29 — DEREALISM

- **Avoid pitfall of infinite gabarit by *symbolic* testing.**

  Recurrence involves a *reduction* of test $n+1$ to test n.
  Second order quantification in Dedekind definition of $\mathbb{N}$.
  Proof-net: existential $\exists X\, A$ involves *witness* T in $A[T]$.
  T is indeed a *synthetic* component of the answer.

- ***Derealism* not Object/Subject: answer partly *subjective*.**

  Épure: combination vehicle + gabarit. Object + *look* at it.

- **Gabarits come by *pairs* $T, \sim T$; are they *balanced*?**

  Gabarit/vehicle: similar to police/yakuza.
  Derealism: some police in the role of yakuza.
  Conflict of interest: gabarit-test has rather be laxist.

- ***Apodictic* (literally, proven): *legitimate* certainty impossible.**

  Epidictic: *reasonable* certainty; belief in *balanced* gabarits.

30 — ONE CAN PREDICT EVERYTHING

- **Third subliminal *transparentist* slogan: negates doubts.**

  Date of death known *in advance*: paradoxical.

  Subjective break: *wrong news* in to-morrow's paper.

  Retrodiction: Nostradamus w.r.t. death of Princess Diana.

- **Prediction in *conditional* tense.**

  Counterfactuals if... Parallel models à la Kripke.

  Conditional premise: stands as *joke* or result of *sake*.

  If I married your mother in 1946, I would now be your *father*.

  Sanma no aji (1962): if Japan *had won* the war, then...

- ***Inverse reasoning* in mathematics yields *conjectures*.**

  Abduction: all conjectures true.

  If $A \Rightarrow B$ then $B \Rightarrow A$.

  Restriction: to be used when it works, i.e., *never*.

  Sherlock Holmes: Conan Doyle selects *relevant* clues.

31 — HEGEL AND CONSISTENCY

- *Paraconsistent:* un-inconsistent, i.e., *undead*.

  Vampires: good for nothing: don't *reflect* in mirrors.
  Adequacy u/u fixed by killing usage: no *consequence*.
  Typical « Theorem »: all integers even and equal to 29.

- **Originates in Brazil**, with plausible influence of *terrorism*.

  Shindô Renmei: paraconsistant *victory* of Japan ($\sim$ 1946).
  Rubber cheque: acceptable only at the point of a gun.

- **Hegel** *mistreated* in xx^{th} century: nazi & paraconsistent.

  Contradictory foundations require answer to *any* question.
  Derealist explanation: épures part of general *animæ*.
  Anima: mingles Object and Subject, cannot be split $\mathcal{V} + \mathcal{G}$.

- 0 admits *animist* proofs: nightmare of *empty* types fixed.

  However $A, \neg A$ cannot both have *non-animist* proofs.

32 — CUT-ELIMINATION

- **Vehicle** $\mathcal{V}$ with conclusions $\vdash \Gamma, [A \otimes \sim A]$ and

  Feedback: $\mathcal{F}_A := [\![\frac{}{p_A(x), p_{\sim A}(x)}]\!]$; fits $p_A(-)$ and $p_{\sim A}(-)$.

  Performance: $\mathcal{V} + \mathcal{F}_A$ possibly yields normal form $\mathcal{W}$.

  Correctness of $\mathcal{W}$ w.r.t. **ordeal** $\mathcal{O}$ for $\vdash \Gamma$.

- **Case** $A = X$: $\mathcal{V} = [\![\frac{}{p_{\sim X'}(x), p_X(x)}]\!] + [\![\frac{}{p_{\sim X}(x), p_{X''}(x)}]\!] + \ldots$.

  Then: $\mathcal{W} = [\![\frac{}{p_{\sim X'}(x), p_{X''}(x)}]\!] + \ldots$ passes test $\mathcal{O}$.

- **Case** $A = B \otimes C$; replace $\mathcal{F}_A$ with $\mathcal{F}_B + \mathcal{F}_C$.

  Change of syntheticity: two cuts $\vdash \Gamma, [B \otimes \sim B], [C \otimes \sim C]$.

  $\mathcal{V} + \mathcal{F}_A$ same normal form as $\mathcal{V} + \mathcal{F}_B + \mathcal{F}_C$.

- **Replacing** $[\![\frac{q_D(x)}{p_D(x)}]\!]$ with $[\![\frac{q_D(x)}{}]\!]$ in $\mathcal{O}$ yields **closing** $\mathcal{O}'$.

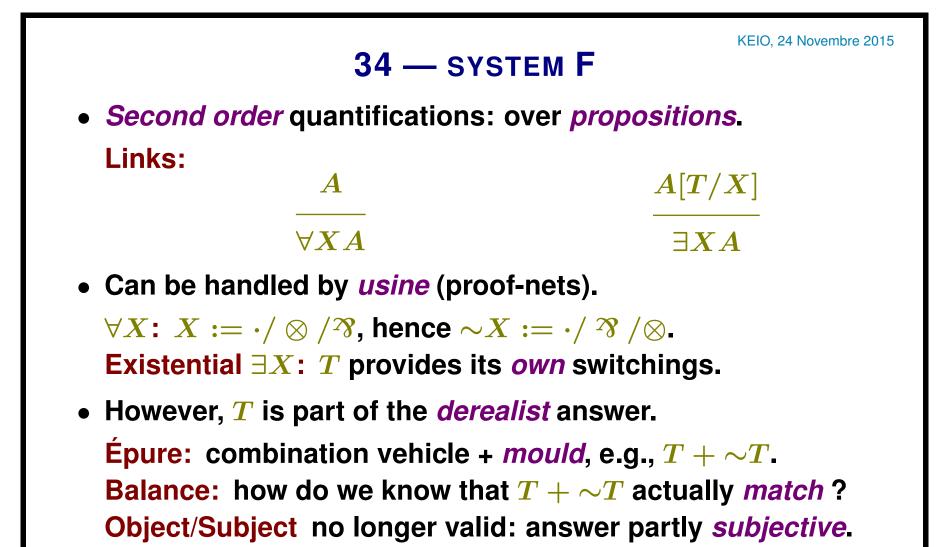 **Main result:** $\mathcal{V} + \mathcal{O}'$ normalises into:

  $[\![\frac{}{p_B(x)}]\!] + [\![\frac{}{p_C(x)}]\!] + [\![\frac{}{p_{\sim B}(x), p_{\sim C}(x)}]\!]$.

33 — EXPONENTIAL CUTS

- **Cut on $A = B \oslash C$.**

  $\mathcal{V} + \mathcal{F}_A$ **same normal form as** $\mathcal{V} + \mathcal{F}_B + \mathcal{F}_C$**.**
  Same as: $\mathcal{V} + \mathcal{F}_B \otimes t_1 + \cdots + \mathcal{F}_B \otimes t_n + \mathcal{F}_C$**.**
  Choice between: $p_B(x) \otimes y := p_B(x) \cdot y / := p_B(x \cdot y)$**?**
  Knitting: **second solution enables change of syntheticity.**

- **Multiplicative cut-elimination works *mutatis mutandis*.**

  Cut **on** A **replaced with several cuts (**C **and copies of** B**).**
  Copies **not well defined: may change with switching.**

- **Problem when *resuming* cut-elimination.**

  Unrelated **switchings of the cuts** $[B \otimes \sim B] \otimes t_i$**.**
  Non-deterministic **sum of all switchings of** $B \otimes \sim B$**.**
  Independence: **when auxiliary parameters** y, y' **distinct.**

34 — SYSTEM F

- *Second order* quantifications: over *propositions*.

  Links:

  $$\frac{A}{\forall X\, A} \qquad\qquad \frac{A[T/X]}{\exists X\, A}$$

- **Can be handled by *usine* (proof-nets).**

  $\forall X\colon X := \cdot/\otimes/\invamp$, hence $\sim X := \cdot/\invamp/\otimes$.
  Existential $\exists X\colon T$ **provides its *own* switchings.**

- **However,** T **is part of the *derealist* answer.**

  Épure: combination vehicle + *mould*, e.g., $T + \sim T$.
  Balance: how do we know that $T + \sim T$ actually *match* ?
  Object/Subject no longer valid: answer partly *subjective*.
  Answer combines *analytic* and *synthetic* features.
  Epidictic: uncheckable affirmation. $\neq$ *apodictic*.

35 — ANIMÆ

- **Derealism: two pairs, `blue`/`yellow` and `red`/`cyan`.**

  Animæ: uses colours `blue`, `red`.

  Épure: splits as $\mathcal{V} + \mathcal{M}$.

  Animist otherwise: Object and Subject *intertwined*.

  Ordeal: uses colours `yellow`, `cyan`, `black`.

- ***Additive neutrals:* no balance problem in $\exists X\, X$.**

  $\top$: unique ordeal $[\![\, \frac{R(x), S(x)}{}\,]\!] + [\![\, \frac{T(x)}{\top(x)}\,]\!]$.

  0: three ordeals, $[\![\, \frac{r(x)}{}\,]\!] + [\![\, \frac{s(x), t(x)}{O(x)}\,]\!]$ and

  $[\![\, \frac{s(x)}{}\,]\!] + [\![\, \frac{r(x), t(x)}{O(x)}\,]\!]$ and $[\![\, \frac{t(x)}{O(x)}\,]\!]$.

- **The absurdity has an *animist* proof:**

  $[\![\, \frac{}{t(x)}\,]\!] + [\![\, \frac{}{r(x), s(x)}\,]\!]$.

  But no épure: hence consistency.

IV — THE BLOODY REALITY

36 — THE EXPULSION OF SUBJECT

- *Subjectivistic* paranoïa: *exagerates* syntheticity.
  Number 13: rename into row 14. Complotism.
  Causality subjectivistic: butterfly cannot *cause* storm.

- *Objectivistic* schizophrenia: *negates* syntheticity.
  Left-handed cups.
  Ptolemaic astronomy: parallax objectivised into *epicycles*.

- Causality, consequence, *subsequence*.
  Cause before *effect*; hence « subsequence ».
  Saint Anthony patron saint of subsequence.

- Causality $\neq$ consequence.
  I am living $\Rightarrow$ I was born. Not a causality!

- *Objectivisation* of consequence.
  Possible worlds: Leibniz equality.

37 — MISERY OF SEMANTICS

- **Semantic dogma of *subsequence*.**

  Consequence reduced to factual justification.

  Aristotle: *paralogism*. However, accepted factual *refutation*.

- **Non euclidian geometries: Euclid's postulate.**

  Sphere: no parallels; *atomic plant*, too many.

  Much better than cognitive investigation; but *accidental*.

- **Deficit of reality with** $\mathbb{N}$.

  No realist explanation of *absence* of consequence.

  Only one universe of integers (*Kronecker*).

  Non standard integers, sorts of *epicycles* of realism.

  Selfy of incompleteness. Should be other way around.

  Not analytic: out of reach, not computable.

- ***Derealism:*** the look at an *object* part of the object.

  Épure: combination vehicle + *mould*.

38 — REALITY AS KNITTING

- *Chicken and egg* dilemma: search for *objectivity*.

  Ohm's law: $U = R \times I$.

  Enables measures of tension, resistance.

  Verifies law through artifact made according to law.

- Abstractions as *ideal* limits: measure of *tension*.

  $U = (r + R) \times I = r \times I + R \times I$; if ratio r/R « small ».

  Actual U obtained as limit $R \to \infty$.

- No hen/egg, only *knitting* constat/performance/usine/usage.

  Reality: the forgetting of the knitting.

- *Realism:* the forgetting of the forgetting.

  Leads to identifications implicit/explicit, analytic/synthetic.

  Alternative refusal of *performance:* pravdameter or

  Usine: (non monotonicity) or

  Usage: (paraconsistency).

39 — ABOUT DOUBT

- **Cannot doubt of *everything*, e.g., that I wrote `wrote`.**

- ***Reasonable* doubts: as to medicine and *empirical* activities.**
  Empirism expresses doubts; but *generates* them!
  Repetitivity: *same* (close) causes yield *same* (close) effects.
  Approximate testing (Popper) not predictive; the *butterfly*.
  Lourdes confirms unreliability of medicine.

- ***Reasonable* certainty: suspended doubts.**
  Legitimate doubts due to *derealistic* features.
  Deductive method replaces *inductive* empirism.
  Understanding: knitted knowledge. *Mathematics*.
  4-colours proof not knitted enough for mathematical taste.
  Quine's `NF` refused because not knitted to mathematics.

- **No knitting *criterion* (cut-elimination, Church-Rosser).**
  Science: only produces the best knitting *so far*.

40 — THE CONTROVERSIAL PREDICATES

- **System $\mathbb{F}$: propositions are (roughly) enough.**
  Forgetful functor: keeps computational (analytic) contents.
  Realisability: awkward reduction predicate $\rightsquigarrow$ proposition.
  Drop in quality when passing from boolean to *cylindric*.

- *Predicate calculus:* XIX$^{\text{th}}$ century legacy.
  Axiomatics: cannot avoid « Barbari » $\forall x A \vdash \exists x A$.
  Semantics: models non-empty; but justification empty.

- **Dubious principle: besides *proper* variables, used for $\vdash \forall$**
  Junk variables: dedicated to the sole *Barbari.*

- **Intrusion of reality through *external* domain.**
  Variables, functions: proceed from the Sky.

- **In constrast to *propositional* quantification:**
  Variables: refer to propositions, well-defined by l'usine.
  Functions: refer to connectives.

41 — EQUALITY

- *Logical* primitive mistreated by metaphysical *axiomatics:*

  E.g., a predicate: « function » individuals $\rightsquigarrow$ propositions.

- And/or through *semantic* pleonasm:

  BHK: empty, reduces proof of $t = u$ to semantics.
  Semantics: $t = u$ true when *same* denotation: $|t| = |u|$.

- $\forall X \,(Xt \Rightarrow Xu)$ (Leibniz) interesting, *since* totally wrong.

  2nd order: not expected at elementary level.
  Circular: are those two « c » equal? Prejudiced:
  Relevant properties: those compatible with. . . equality.

- A logical *epicycle*, i.e., a realistic contraption.

  Individuals + predicates: *all* of those which are *relevant.*

- Break epicycle by replacing *individual* t with *proposition* t.

  Meaning: « I am t ». Equality as logical equivalence $t \equiv u$.

42 — INDIVIDUALS AS MULTIPLICATIVES

- *Individuals = proposition* forbidden by prejudice:

  Classical: $t \equiv u \vee u \equiv v \vee v \equiv t$. **Only two individuals.**

  Intuitionistic: $\neg\neg(t \equiv u \vee u \equiv v \vee v \equiv t)$. **Not more than** 2.

  Linear: with $(t \multimap u) \mathbin{\&} (u \multimap t)$ **as equality. No obstacle.**

- n-**ary multiplicative: sets of partition of** $\{1, \ldots, n\}$.

  Duality: $\mathcal{C} \perp \mathcal{D}$ **iff their incidence graph is a tree** $(n \neq 0)$.

  Multiplicative: **non-trivial set of partitions equal to bidual.**

  Example: $\otimes := \{\{1, 2\}\}$ **vs.** $\invamp := \{\{1\}, \{2\}\}$.

  Series/parallel: $\P := \{\{1, 2\}, \{3, 4\}\} + \{\{2, 3\}, \{4, 1\}\}$.

  Not sequential: $\P$ **admits proof-nets, no sequent calculus.**

- *Linear* **implication between multiplicatives:**

  Same $n :$ **typically,** $* \otimes (* \invamp *) \multimap (* \otimes *) \invamp *$ **with** $n = 3$.

  $\sharp$ **partitions: decreases; equal in case of equivalence.**

  Equality: equivalence yields two *isomorphisms*, **not related.**

43 — FUNCTIONS AND PREDICATES

- **Functional *terms* come from same multiplicative matrix:**

  Positive multiplicatives with possible repetitions.

  Example: $x \,\invamp\, (x \otimes y)$**. No constant, no *Barbari*, no regrets.**

  Pairing: ensured by $(x \,\invamp\, y) \otimes (x \,\invamp\, x \,\invamp\, y)$.

- ***Predicate* variables $P, Q, \ldots$ as variable *connectives.***

  Pt **handled by unknown binary connective K.**

  Usage: all possible uses $Kt\tilde{t}$ of individual t and negation $\tilde{t}$.

  Usine: enough to test with $K = \otimes$ and $K = \invamp$.

  Equality principle: $t = u \Rightarrow (Pt \multimap Pu)$ OK'ed by l'usine.

  Refused: $t = u \Rightarrow (Pt \multimap Qu)$ **and** $t = u \multimap (Pt \multimap Pu)$.

- ***Equality* handled by:** $(\tilde{t} \,\invamp\, u) \,\&\, (t \,\invamp\, \tilde{u})$.

- **First-order quantification: restriction of « full » case.**

  Existential witnesses: taken among multiplicative terms.

44 — DISCUSSION

- **Logic is second order, including so-called first-order:**

  Propositions: variables, implicit $\forall X$ performed after.
  Usage: externalised by counter-models ($\exists X$ forbidden).
  No testing: dubious advantage of externalisation.

- **Individuals:** *tame* **second order. No derealism.**

  Witnesses: multiplicatives, limited loss of subformula pty.
  Balance: rights/duties, usine/usage not really problematic.

- *Arithmetic:* **all axioms removed but:**

  Third/fourth Peano axioms: $Sx \neq 0$ and $Sx = Sy \Rightarrow x = y$.

- **The origin of logical doubt (incompleteness, etc.):**

  Épure vs. gabarit: performance $\mathcal{V} + \mathcal{M} + \mathcal{G}$.
  Variance: usine works better with lax $\mathcal{M}$. Usage may fail.
  Example: induction on « ill-formed » $\mathcal{M}$.

45 — ANTI-CLASSICAL PROGRAM

- **Idea: sever all *bridges* with semantics.**

  Refute classical principles, e.g., weakening/contraction.
  $\neg \forall X \forall Y ((X \otimes Y) \Rightarrow X)$ and $\neg \forall X (X \Rightarrow (X \otimes X))$.

- **Expected outcome: increase in *logical* expressiveness.**

  Natural numbers: complete *logicisation* of arithmetic.
  $\overline{m} \neq \overline{n}$ (for $m \neq n$) not provable in linear logic.

- **Unfortunate « classical » forgetful functor.**

  Clue: use *non sequential* connectives, e.g., ¶.
  Semantics: inexistant. Indeed, intersection types.

  $\P(A, B, C, D) = ((A \otimes B) \,\mathfrak{P}\, (C \otimes D)) \cap ((B \otimes C) \,\mathfrak{P}\, (D \otimes A))$.

  $\sim\!\P(\sim\!A, \sim\!B, \sim\!C, \sim\!D) = ((A \otimes C) \,\mathfrak{P}\, (B \,\mathfrak{P}\, D)) \cap ((A \,\mathfrak{P}\, C) \,\mathfrak{P}\, (B \otimes D))$.

- **Conjecture: find a classical inconsistent multiplicative.**